

## 第五章限时练习答案

一, 选择题

1-5 BBBAD    6—10DBCBD

二, 填空题

1,  $\frac{1}{2e}$     2,  $e^{\sin xy} \cdot \cos xy (ydx + xdy)$     3, 0    4,  $\frac{yz}{e^z - xy}$     5, -5

三, 计算题

1, 解:  $\frac{\partial z}{\partial x} = \frac{e^x}{e^x + e^y}$      $\frac{\partial z}{\partial y} = \frac{e^y}{e^x + e^y}$

$$\frac{\partial^2 z}{\partial x^2} = \frac{e^{x+y}}{(e^x + e^y)^2} \quad \frac{\partial^2 z}{\partial y^2} = \frac{e^{x+y}}{(e^x + e^y)^2}$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y} = \frac{-e^{x+y}}{(e^x + e^y)^2}$$

2, 解: 令  $u = x^2 - e^y$      $v = \frac{x}{y}$

$$\therefore \frac{\partial z}{\partial x} = f'_u \cdot 2x + f'_v \cdot \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = f'_u \cdot (-e^y) + f'_v \cdot \left(-\frac{x}{y^2}\right)$$

$$\frac{\partial^2 z}{\partial x \partial y} = -[f''_{uv} \cdot 2xe^y + f''_{uv} \cdot \frac{2x^2 + ye^y}{y^2} + f''_{vv} \cdot \frac{x}{y^3} + f'_v \cdot \frac{1}{y^2}]$$

3, 解:

令  $F(x, y, z) = xyz - e^z$

$$\therefore \frac{\partial z}{\partial x} = -\frac{F'_x}{F'_z} = \frac{yz}{e^z - xy} \quad \frac{\partial z}{\partial y} = -\frac{F'_y}{F'_z} = \frac{xz}{e^z - xy}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial \left(\frac{\partial z}{\partial x}\right)}{\partial y} = \frac{\partial \left(\frac{yz}{e^z - xy}\right)}{\partial y} = \frac{z}{e^z - xy} + \frac{2xyz}{(e^z - xy)^2} - \frac{e^z xyz^2}{(e^z - xy)^3}$$

4, 解:

$$\text{令 } F(x, y, z) = e^z - z + xy - 4$$

$$\therefore \vec{n} = \vec{s} = \{F'_x, F'_y, F'_z\}|_{(1,3,0)} = \{y, x, e^z - 1\}|_{(1,3,0)} = \{3, 1, 0\}$$

$$\therefore \text{直线方程为: } \frac{x-1}{3} = \frac{y-3}{1} = \frac{z}{0}$$

$$\text{平面方程为: } 3x + y - 6 = 0$$

5 解:

$$\text{令 } L(x, y, \lambda) = x - 2y + \lambda(x^2 + y^2 - 1)$$

$$\therefore \begin{cases} L'_x = 0 \Rightarrow 1 + 2x\lambda = 0 \\ L'_y = 0 \Rightarrow -2 + 2y\lambda = 0 \\ L'_\lambda = 0 \Rightarrow x^2 + y^2 = 1 \end{cases}$$

$$\therefore \text{极值点为: } \left(\frac{\sqrt{5}}{5}, -\frac{2\sqrt{5}}{5}\right) \text{ 与 } \left(-\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}\right)$$

$$\text{则极值为: } z_{\text{极大值}} = \sqrt{5}, z_{\text{极小值}} = -\sqrt{5}$$